

Recoil Order Chiral Corrections to Baryon Octet Axial Currents

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Abstract

We calculate chiral corrections to the octet axial currents through $\mathcal{O}(p^3)$ using baryon chiral perturbation theory (BCPT). The relativistic BCPT framework allows one to sum an infinite series of recoil corrections at a given order in the chiral expansion. We also include SU(3)-breaking operators occurring at $\mathcal{O}(p^2)$ not previously considered. We determine the corresponding low-energy constants (LEC's) from hyperon semileptonic decay data using a variety of infrared regularization schemes. We find that the chiral expansion of the axial currents does not display the proper convergence behavior, regardless of which scheme is chosen. We explore the implications of our analysis for determinations of the strange quark contribution to the nucleon spin, Δs .

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I. INTRODUCTION

In the pseudoscalar meson sector chiral perturbation theory (CPT) provides a consistent and systematic framework for calculating physical observables. Generally, they can be expanded order by order in powers of p/Λ_χ , where $\Lambda_\chi = 4\pi F_\pi$, $F_\pi = 92.4\text{MeV}$, and p is the typical small momenta or mass of the Goldstone bosons. When CPT is extended to include the baryons, a difficulty arises due to the presence of the large baryon mass. One encounters terms like m_N/Λ_χ , which obscure the convergence of the chiral expansion. To overcome this difficulty, heavy baryon chiral perturbation theory (HBCPT) was introduced [3,4,6]. In this non-relativistic framework, the baryon mass appears only in vertices as powers of $1/m_N$. One thus obtains a consistent expansion in two small parameters, p/m_N and p/Λ_χ . This approach has been applied successfully to a wide variety of baryon observables.

Despite its successes, HBCPT comes with its own shortcomings. For example, the $1/m_N$ corrections are unnaturally large for some observables. In some cases, one requires a large number of higher-order terms in $1/m_N$ in order to obtain proper convergence behavior of the chiral expansion. From a conceptual standpoint, it was noted in [1] that HBCPT fails to produce the correct analytical structure near threshold for the nucleon isovector electromagnetic form factors. The underlying reason is the non-relativistic treatment of the baryon propagators in HBCPT.

A relativistic formulation of CPT for baryons was recently proposed in Ref. [5] and applied to nucleon electromagnetic form factors in Ref. [7]. This formulation, which we denote BCPT (baryon CPT), circumvents the problematic $(m_N/\Lambda_\chi)^n$ terms by splitting a given chiral loop integral into an infrared sensitive term, I , and an infrared insensitive, or “regular” piece, R . The former contains all the non-analytic contributions uniquely identified with chiral loops; the latter contains the power dependence on baryon mass. Since R is also analytic in quark masses, its contribution may be completely absorbed into the appropriate terms in a chiral Lagrangian. Since the corresponding low-energy constants (LEC’s) are determined entirely from fits to experimental data, the $(m_N/\Lambda_\chi)^n$ behavior never appears explicitly. Moreover, by retaining the fully relativistic form of the baryon propagators in I , one includes all of the recoil corrections to the non-analytic contributions at a given order in the chiral expansion. This procedure, known as “infrared regularization”, contrasts with the HBCPT approach, where one must explicitly work out the recoil corrections order-by-order in $1/m_N$.

The simplifications introduced by BCPT have been explored in the case of a few observables. In Ref. [7], for example, it was pointed out that BCPT improves the convergence of the chiral expansion of the nucleon electromagnetic form factors as compared to HBCPT. Moreover, since BCPT is relativistic, analytical behaviour of the resulting form factors is correct.

In this work we employ BCPT to calculate the one-loop chiral corrections to the axial currents of the octet baryons. The leading order chiral corrections to the axial currents are of the form $m_s \ln m_s$ and were first calculated in [2]. For a subtraction scale of $\mu = 1\text{ GeV}$ the corrections calculated in [2] are less than 30%. However, the correction due to the wave function renormalization was ignored in [2] as pointed out in [3], where the same problem was treated with HBCPT formalism. When wave function renormalization and vertex corrections are both included, the leading one-loop correction is large [3]. For example, the fit values of

the $SU(3)$ couplings at tree level in Ref. [3] are $D = 0.80 \pm 0.14$, $F = 0.50 \pm 0.12$. The one-loop chiral corrections shifted the best fit to $D = 0.56 \pm 0.1$, $F = 0.33 \pm 0.06$ [3]. Later the same authors included the intermediate decuplet baryon states in the chiral loops and found significant cancellations with the octet contributions [4]. While this cancellation suggests the importance of including the decuplet for obtaining proper convergence, inclusion of the decuplet is not sufficient in the case of some other observables. In the case of octet baryon magnetic moments, for example, one must also include the leading recoil-order ($1/m_N$) corrections [8].

In this paper, we use BCPT to explore the effect of recoil corrections on the convergence of the chiral expansion of the octet axial currents. We also include $\mathcal{O}(p^2)$ chiral symmetry breaking terms not included in previous analyses. In order to maintain predictive power, we truncate the expansion at $\mathcal{O}(p^3)$. The number of LEC's appearing at $\mathcal{O}(p^4)$ prevents one from carrying out a model independent analysis. We also follow Refs. [2,3] and set $m_u = m_d = 0$ in performing numerical fits, although the formulae presented below included results for non-vanishing pion mass. We find that the impact of the $\mathcal{O}(p^2)$ symmetry breaking (SB) operators is noticeable. More importantly, the $\mathcal{O}(p^3)$ contributions – corresponding entirely to loop-generated recoil corrections – are generally larger than the $\mathcal{O}(p^2)$ terms. Thus, the chiral expansion of the axial currents does not converge in the manner expected when decuplet contributions are integrated out. While the significance of the $\mathcal{O}(p^2)$ loop corrections was first noted in Ref. [3], our study of the expansion through $\mathcal{O}(p^3)$ makes the non-convergence of the series abundantly clear. Contrary to one's naïve hope, inclusion of octet-only recoil order contributions only worsens the convergence properties of the axial currents. Whether explicit inclusion of recoil order decuplet contributions remedies this situation remains to be seen.

In a related issue, the definition of the infrared loop contributions contains a degree of ambiguity. While the non-analytic quark mass dependence of I is unique, this integral may also contain terms analytic in m_q . Whether or not one retains these analytic contributions explicitly is a matter of convention. The standard practice in HBCPT is to keep only the non-analytic loop effects. On the other hand, the authors of Refs. [5,7] also retain analytic pieces of I . We analyze the axial currents using both schemes. In this case, the difference amounts only to the treatment of $\mathcal{O}(p^2)$ analytic contributions to wavefunction renormalization. The corresponding impact on the convergence properties of the chiral expansion is small. We cannot, however, determine whether this scheme insensitivity persists to higher order. While the integrals I for the axial currents contain a variety of $\mathcal{O}(p^4)$ and higher contributions, we truncate at $\mathcal{O}(p^3)$ for reasons noted earlier.

II. INFRARED REGULARIZATION

The motivation and formalism for BCPT are explained extensively in Refs. [5,7]. Interested readers may consult these two references for details. The key feature of BCPT is the so-called infrared regularization procedure. Following Refs. [5,7] we illustrate using the one-loop baryon self energy. The ultraviolet (UV) divergence of the one-loop integral is regulated using dimensional regularization. The regulated integral H is then separated into the I and R pieces using Feynman parameters:

$$\begin{aligned}
H &= -i \int \frac{d^d k}{(2\pi)^d} \frac{1}{AB} = -i \int_0^1 dz \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(1-z)A + zB]^2} \\
&= -i \left\{ \int_0^\infty - \int_1^\infty \right\} dz \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(1-z)A + zB]^2} = I + R,
\end{aligned} \tag{1}$$

with $A = M^2 - k^2 - i\epsilon$, $B = m_N^2 - (p - k)^2 - i\epsilon$ and M, m_N is the pseudoscalar and nucleon mass. The region of parameter integration for the integral I contains $z = 0$. At the origin, the denominator is proportional to A^2 , and thus, is highly infrared sensitive (singular in the case of the self energy). As shown in Ref. [5], all of the non-analytic m_q -dependence uniquely associated with the loop is contained in I . For the regular part, R , the Feynman parameter runs from one to infinity, and the result is analytic. Consequently, its contribution can be entirely absorbed into the appropriate operators appearing in the effective Lagrangian. In addition, if we expand I in terms of $1/m_N$, we recover the HBCPT result at each order. Thus, the infrared sensitive part I of the corresponding relativistic diagram is just the sum of the leading HBCPT diagram and diagrams with $1/m_N$ insertions to all orders. In other words, BCPT effectively sums the $1/m_N$ series in HBCPT.

It is important to note that the inclusion of the full tower of recoil corrections renders the chiral counting somewhat ambiguous. Contributions involving recoil effects have the generic form

$$\frac{m_X^2}{\Lambda_X^2} \mu^k f(\mu) \quad , \tag{2}$$

where m_X is a pseudoscalar mass, $\mu = m_X/m_N \propto \sqrt{m_q}/m_N$, and $f(\mu)$ is a recoil factor. For the axial currents, $\mu^k f(\mu)$ is non-analytic in m_q and, therefore, can never be generated by terms in the effective Lagrangian. Nevertheless, one may perform a Taylor expansion of $f(\mu)$ in powers of μ^2 about $\mu^2 = 0$. Consequently, the quantity in Eq. (2) contains an infinite series of contributions of successively higher orders in p . The first term in the series – obtained by replacing $f(\mu) \rightarrow f(0)$ – is purely of $\mathcal{O}(p^{k+2})$. In the language of HBCPT, this first term in the series constitutes its leading contribution in the $1/m_N$ expansion. In what follows, we identify the chiral order of the term in (2) by the order of its leading term in the $1/m_N$ expansion.

We emphasize that the chiral order of the recoil term in (2) is unambiguous only in the heavy baryon limit. Retention of the higher order terms associated with recoil factors is the price one must pay for maintaining the analyticity properties of loops implied by relativity and crossing symmetry. These properties are lost in HBCPT. It does not appear possible to respect the full analytic structure of chiral loops and maintain the standard chiral counting procedure simultaneously. Fortunately, in the case of the axial currents, we find that the *numerical* impact of setting $f(\mu) \rightarrow f(0)$ is negligible. In short, it is sufficient to work to $\mathcal{O}(1/m_N)$ in the heavy baryon expansion in order to ascertain the effects of recoil.

It was also argued in Ref. [5] that the baryon mass m_N serves as a “natural” subtraction scale in BCPT using the infrared regularization. We follow this convention and set this subtraction scale equal to m_N .

III. AXIAL CURRENTS

In writing down the octet axial currents, we follow standard conventions and notations. The most general meson-baryon Lagrangian at lowest order is

$$\begin{aligned}\mathcal{L}_0 = & i \operatorname{Tr} \left(\bar{B} (\gamma^\mu D_\mu - m_N) B \right) + D \operatorname{Tr} \left(\bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} \right) + F \operatorname{Tr} \left(\bar{B} \gamma^\mu \gamma_5 [A_\mu, B] \right) \\ & + \frac{F_\pi^2}{4} \operatorname{Tr} \left((D^\mu \Sigma)^\dagger D_\mu \Sigma \right) + a \operatorname{Tr} M (\Sigma + \Sigma^\dagger),\end{aligned}\quad (3)$$

where

$$D_\mu B = \partial_\mu B + [V_\mu, B], \quad (4)$$

$$V_\mu = \frac{1}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi) \quad (5)$$

$$A_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi) \quad (6)$$

$$\xi = e^{i \frac{\pi}{F_\pi}}, \quad \Sigma = \xi^2 = e^{2i \frac{\pi}{F_\pi}}, \quad (7)$$

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix} \quad (8)$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix} \quad (9)$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad (10)$$

One may obtain vector and axial vector current operators from \mathcal{L}_0 by including vector and axial vector sources in the covariant derivatives. The leading $\mathcal{O}(p^0)$ operator contains only baryon fields and the LEC's D and F . Axial currents involving both baryons and mesons first appear at $\mathcal{O}(p)$. Additional purely baryonic axial currents appear at $\mathcal{O}(p^2)$. They arise from the SU(3) SB Lagrangian

$$\begin{aligned}\mathcal{L}_1 = & \frac{m_K^2}{\Lambda_\chi^2} \{ d_1 \operatorname{Tr} \left(\bar{B} \gamma^\mu \gamma_5 \{A_\mu, \chi_+\} B \right) + d_2 \operatorname{Tr} \left(\bar{B} \gamma^\mu \gamma_5 A_\mu B \chi_+ \right) \\ & + d_3 \operatorname{Tr} \left(\bar{B} \gamma^\mu \gamma_5 \chi_+ B A_\mu \right) + d_4 \operatorname{Tr} \left(\bar{B} \gamma^\mu \gamma_5 B \{A_\mu, \chi_+\} \right) \},\end{aligned}\quad (11)$$

where

$$\chi_+ = \frac{1}{2}(\xi^+ \chi \xi^+ + \xi \chi^+ \xi) \quad (12)$$

$$\chi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (13)$$

The LECs d_{1-4} are expected to be order of unity in our normalization. There are two other terms involving χ :

$$\text{Tr} \left(\bar{B} \gamma^\mu \gamma_5 [A_\mu, B] \right) \text{Tr}(\chi_+) \quad , \quad \text{Tr} \left(\bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} \right) \text{Tr}(\chi_+). \quad (14)$$

These terms do not break $SU(3)$ symmetry and can be absorbed into the definition of D, F terms in Eq. (3).

Using $\mathcal{L}_{0,1}$ one obtains the axial current:

$$\begin{aligned} J_\mu^A = & \frac{1}{2} D \text{Tr} \left(\bar{B} \gamma_\mu \gamma_5 \{ \xi T^A \xi^\dagger + \xi^\dagger T^A \xi, B \} \right) \\ & + \frac{1}{2} F \text{Tr} \left(\bar{B} \gamma_\mu \gamma_5 [\xi T^A \xi^\dagger + \xi^\dagger T^A \xi, B] \right) \\ & + \frac{1}{2} d_1 \frac{m_K^2}{\Lambda_\chi^2} \text{Tr} \left(\bar{B} \gamma^\mu \gamma_5 \{ \xi T^A \xi^\dagger + \xi^\dagger T^A \xi, \chi_+ \} B \right) \\ & + \frac{1}{2} d_2 \frac{m_K^2}{\Lambda_\chi^2} \text{Tr} \left(\bar{B} \gamma^\mu \gamma_5 (\xi T^A \xi^\dagger + \xi^\dagger T^A \xi) B \chi_+ \right) \\ & + \frac{1}{2} d_3 \frac{m_K^2}{\Lambda_\chi^2} \text{Tr} \left(\bar{B} \gamma^\mu \gamma_5 \chi_+ B (\xi T^A \xi^\dagger + \xi^\dagger T^A \xi) \right) \\ & + \frac{1}{2} d_4 \frac{m_K^2}{\Lambda_\chi^2} \text{Tr} \left(\bar{B} \gamma^\mu \gamma_5 B \{ \xi T^A \xi^\dagger + \xi^\dagger T^A \xi, \chi_+ \} \right) \\ & + \frac{1}{2} \text{Tr} \left(\bar{B} \gamma_\mu [\xi T^A \xi^\dagger - \xi^\dagger T^A \xi, B] \right) \\ & + \frac{i}{2} F_\pi^2 \text{Tr} T^A \left((\partial_\mu \Sigma)^\dagger \Sigma - \partial_\mu \Sigma \Sigma^\dagger \right). \end{aligned} \quad (15)$$

Renormalized matrix elements of J_μ^A between octet baryon states may be written as

$$\langle B_i | J_\mu^A | B_j \rangle = \{ \alpha_{ij} + \bar{\alpha}_{ij} \frac{m_K^2}{\Lambda_\chi^2} + [\lambda_{ij}^\pi I_d^\pi + \lambda_{ij}^K I_d^K + \lambda_{ij}^\eta I_d^\eta] \alpha_{ij} \quad (16)$$

$$\begin{aligned} & + [\beta_{ij}^\pi I_a^\pi + \beta_{ij}^K I_a^K + \beta_{ij}^\eta I_a^\eta] + [\gamma_{ij}^\pi I_b^\pi + \gamma_{ij}^K I_b^K + \gamma_{ij}^\eta I_b^\eta] \alpha_{ij} \\ & + [\theta_{ij}^\pi I_c^\pi + \theta_{ij}^K I_c^K + \theta_{ij}^\eta I_c^\eta] \alpha_{ij} \} \bar{u}_{B_i} \gamma_\mu \gamma_5 u_{B_j} \end{aligned} \quad (17)$$

where the first term on the right hand side is the lowest order one. In terms of the D and F coefficients, this term reads for different octet states

$$\begin{aligned} \alpha_{pn}^{1+i2} &= (D + F) , \\ \alpha_{\Lambda\Sigma^-}^{1+i2} &= \frac{2}{\sqrt{6}} D , \\ \alpha_{\Xi^0\Xi^-}^{1+i2} &= (D - F) , \end{aligned}$$

$$\begin{aligned}
\alpha_{p\Lambda}^{4+i5} &= -\frac{1}{\sqrt{6}}(D+3F) , \\
\alpha_{\Lambda\Sigma^-}^{4+i5} &= -\frac{1}{\sqrt{6}}(D-3F) , \\
\alpha_{n\Sigma^-}^{4+i5} &= (D-F) , \\
\alpha_{\Sigma^0\Sigma^-}^{4+i5} &= \frac{1}{\sqrt{2}}(D+F) = \sqrt{2}\alpha_{\Sigma^+\Xi^0}^{4+i5}
\end{aligned} \tag{18}$$

where the superscript denotes the corresponding SU(3) indices of the current.

The second term arises from the SB terms in Eq. (11). The coefficients $\bar{\alpha}_{ij}$ are

$$\begin{aligned}
\bar{\alpha}_{pn}^{1+i2} &= d_2 , \\
\bar{\alpha}_{\Lambda\Sigma^-}^{1+i2} &= 0 , \\
\bar{\alpha}_{\Xi^0\Sigma^-}^{1+i2} &= d_3 , \\
\bar{\alpha}_{p\Lambda}^{4+i5} &= -\frac{1}{\sqrt{6}}(d+3f+2d_2) , \\
\bar{\alpha}_{\Lambda\Sigma^-}^{4+i5} &= -\frac{1}{\sqrt{6}}(d-3f+2d_3) , \\
\bar{\alpha}_{n\Sigma^-}^{4+i5} &= (d-f) , \\
\bar{\alpha}_{\Sigma^0\Sigma^-}^{4+i5} &= \frac{1}{\sqrt{2}}(d+f) = \sqrt{2}\bar{\alpha}_{\Sigma^+\Xi^0}^{4+i5}
\end{aligned} \tag{19}$$

where

$$d = \frac{d_1 + d_4}{2}, \quad f = \frac{d_1 - d_4}{2}. \tag{20}$$

The remaining terms arise from the loops of Figs 1 and 2. The coefficients λ_{ij}^X , β_{ij}^X , γ_{ij}^X and θ_{ij}^X are given in Tables I-III and Eq. (28) below. In presenting the loop results, we give complete expressions for the infrared integrals I . When fitting the LEC's D , F , and d_1, \dots, d_4 , however, we include only the pieces occuring through $\mathcal{O}(p^3)$. In doing so, we follow approach used in Ref. [5] in making the chiral expansion of the loop integrals. The denominator is always kept intact while we expand the numerator up to order $\mathcal{O}(p^3)$ only after finishing the loop integral explicitly. Meanwhile we never expand terms like

$$\left(4 - m_K^2/m_N^2\right)^{\pm 1/2} \tag{21}$$

in order to preserve the analyticity properties of the integrals.

Wavefunction renormalization

The third term in Eq. (16) arises from the wave function renormalization in Fig. 1. We have

$$\begin{aligned}
Z_i &= 1 + \lambda_{ii}^\pi I_d^\pi + \lambda_{ii}^K I_d^K + \lambda_{ii}^\eta I_d^\eta , \\
\sqrt{Z_i Z_j} &= 1 + \lambda_{ij}^\pi I_d^\pi + \lambda_{ij}^K I_d^K + \lambda_{ij}^\eta I_d^\eta
\end{aligned} \tag{22}$$

The coefficients λ_{ij}^X ($X = \pi, K, \eta$) are collected in Table I. The function I_D^X is defined as

$$I_d^X = \frac{1}{4} \{ 4m_N^2 m_X^2 J_A^X(0) - \Delta_X - 2m_X^2 [I^X(m_N^2) + m_X^2 J_A^X(0)] \} \quad (23)$$

where the expressions of $J_A^X(0)$, Δ_X , $I^X(m_N^2)$ are:

$$\Delta_X = \left(\frac{m_X}{\Lambda_X} \right)^2 \log \mu^2 \quad , \quad (24)$$

$$I^X(m_N^2) = \frac{\mu}{\Lambda_X^2} \left\{ -\mu \log \mu + \frac{\mu}{2} - \sqrt{4 - \mu^2} \arccos \left(-\frac{\mu}{2} \right) \right\} \quad , \quad (25)$$

$$J_A^X(0) = \frac{1}{m_N^2 \Lambda_X^2} \left\{ -\log \mu - \frac{1}{2} + \mu \frac{\arccos \left(-\frac{\mu}{2} \right)}{\sqrt{4 - \mu^2}} \right\} \quad , \quad (26)$$

where $\mu = m_X/m_N$.

Vertex corrections

The fourth term comes from the vertex correction diagram Fig. 2a. The coefficients β_{ij}^X are collected in Table II. The function I_a^X is defined as

$$I_a^X = -\Delta_X - m_X^2 I^X(m_N^2) + m_X^4 J_A^X - 2 \frac{m_X^2}{\Lambda_X^2} + \frac{m_X^4}{m_N^2 \Lambda_X^2}$$

where the last two terms arise from expanding the factors $\frac{D-4}{D-2}$, $\frac{1}{D-2}$ around $D = 4$ in the scalar integrals in the appendix of Ref. [7].

The fifth term is the vertex correction from the tadpole diagram in Fig. 2b. The coefficients γ_{ij}^X are presented in Table III. The function $I_b^X = \Delta_X$.

Seagull graphs

The last term in Eq. (16) arises from the $\mathcal{O}(p)$ one-meson operators in J_μ^A . The relevant Feynman diagrams are Fig. 2c and 2d. The contribution from these diagrams is entirely of recoil order, vanishing in the $m_N \rightarrow \infty$ limit. It was not included in the previous HBCPT analyses which worked to leading order in the $1/m_N$ expansion. For this contribution, we have

$$I_c^X = -\frac{1}{2} m_X^2 I^X(m_N^2) \quad (27)$$

$$\theta_{ij}^X = -4 \gamma_{ij}^X \quad (28)$$

with $X = K, \eta$.

It is straightforward to verify that we recover previous results in Refs. [2,3] if we use the relation $m_\eta^2 = \frac{4}{3} m_K^2$ and keep only $m_K^2 \ln m_K^2$ terms in Eq. (16).

IV. NUMERICAL ANALYSIS AND DISCUSSIONS

From the expressions of $I_{a,b,c,d}^X$ we find that the full one loop result Eq. (16) contains terms of $\mathcal{O}(p^3)$ through $\mathcal{O}(p^5)$. The terms of odd chiral order (p^3 , p^5) are entirely non-analytic, whereas the loops yield both analytic and non-analytic contributions of even chiral order (p^2 , p^4). Recoil order contributions first occur at $\mathcal{O}(p^3)$. The contributions at this order have the form given in (2) with $k = 1$ and

$$f(\mu) = \arccos\left(-\frac{\mu}{2}\right) \times (4 - \mu^2)^{\pm 1/2} . \quad (29)$$

Although the $f(\mu)$ is non-analytic in the complex plane, one may nevertheless expand it in powers of μ^2 about $\mu^2 = 0$ along the real axis. When multiplied by the prefactor $(m_X/\Lambda_\chi)^2 \times \mu$ of (2), the leading term in the series scales as $m_X^3/\Lambda_\chi^2 m_N$, making it of chiral order p^3 . The remaining terms – corresponding to successively higher orders in p – represent sub-leading, one-loop recoil corrections. We note, however, that this series cannot be reproduced by any combination of operators in the effective Lagrangian. Each term in the series scales as an odd power of m_X , that is, as a fractional power of quark mass. Thus, the infinite series of recoil corrections given by the factors in Eq. (29) is unambiguously associated with loops. In HBCPT, one would compute these sub-leading corrections order by order in $1/m_N$ and would be forced to truncate at some order.

Loop contributions involving even powers of p cannot be disentangled from terms in the effective Lagrangian. For example, both the wavefunction renormalization diagrams and the vertex corrections generate analytic contributions of $\mathcal{O}(p^2)$. Since these contributions are quadratic in m_X , one could just as well absorb them into the SB terms of Eq. (11). Similarly, at $\mathcal{O}(p^4)$, one encounters a new set of SB contributions generated by the Lagrangian

$$\begin{aligned} \mathcal{L}_2 = & \frac{m_K^4}{\Lambda_\chi^4} \{ d_5 \text{Tr} (\bar{B} \gamma^\mu \gamma_5 \chi_+ A_\mu \chi_+ B) + d_6 \text{Tr} (\bar{B} \gamma^\mu \gamma_5 A_\mu B \chi_+^2) \\ & + d_7 \text{Tr} (\bar{B} \gamma^\mu \gamma_5 \chi_+ B \{A_\mu, \chi_+\}) \} + d_8 \text{Tr} (\bar{B} \gamma^\mu \gamma_5 A_\mu B A_\nu A^\mu) + \dots \end{aligned} \quad (30)$$

where we have included only a few of the relevant terms. Since the number of LEC's appearing at this order is larger than the number of available data, we truncate at $\mathcal{O}(p^3)$ in order to avoid introducing model assumptions.

To this order, we determine the LEC's D , F and d_1, \dots, d_4 from hyperon semileptonic decay data [9], presented in terms of axial vector couplings in Table IV. Since $m_u \sim m_d \ll m_s$, we set $m_\pi = 0$ in performing our numerical fits. We also follow Ref. [3] and enhance the errors by 0.2 to avoid the biasing the fit to the precisely known $D + F$ value from neutron beta decay. The tree level best fit yields $D = 0.78$, $F = 0.47$ and $F/D = 0.60$ with a $\chi^2 = 0.1$ for six data points as presented in Table IV. In obtaining fits at $\mathcal{O}(p^2)$ and beyond, we follow two different procedures for treating the analytic loop terms: scheme B, in which all the $\mathcal{O}(p^2)$ analytic loop contributions are kept explicitly, as in Refs. [5,7]; and scheme C in which these analytic terms are effectively absorbed into the $\mathcal{O}(p^2)$ LEC's.

It is interesting first to truncate at $\mathcal{O}(p^2)$ and determine the impact of the SB contributions. In Ref. [3], where these terms were omitted, the best fit values for the LEC's are $D = 0.56$ and $F = 0.33$ with $F/D = 0.6$. Inclusion of the SB terms shifts these values to

$D = 0.55$ and $F = 0.41$ ($D = 0.51$, $F = 0.37$) in scheme B (C), a 10-25% shift. Similarly, in when the recoil corrections are included but SB terms omitted, the values for D and F are both reduced by roughly 30% from results in Ref. [3] (see Tables V and VII). The full $\mathcal{O}(p^3)$ results yield values of D and F nearly 25% smaller, with F/D remaining close to 0.6. The dominant effect arises from inclusion of recoil.

It is also instructive to determine the numerical importance of including the full recoil factors appearing in Eq. (29). To that end, we make the replacements

$$\arccos\left(-\frac{\mu}{2}\right) \rightarrow \frac{\pi}{2} \quad (31)$$

$$\sqrt{4 - \mu^2} \rightarrow 2 \quad (32)$$

which amounts to retaining only the leading $1/m_N$ corrections appearing at $\mathcal{O}(p^3)$. Taking this limit is equivalent to working to first order in $1/m_N$ with HBCPT. In this case, the best fit values for D and F are essentially unchanged from the $\mathcal{O}(p^3)$ fit for scheme C, while the SB LEC's d_1, \dots, d_4 shift somewhat. Since the impact of the SB terms relative to the recoil corrections is small, it appears that retention of the leading recoil corrections is sufficient to determine the convergence behavior of the expansion.

More significantly, the $\mathcal{O}(p^3)$ contributions, arising entirely from recoil effects, are as large if not larger than the $\mathcal{O}(p^2)$ contributions. The relative importance of each order is shown in Tables VI and VIII. To illustrate, we write here the results for a few representative channels (in scheme C):

$$g_{np}^A = 0.658[1 + 0.419 + 0.495] = 1.26 \quad (33)$$

$$g_{p\Lambda}^A = -0.488[1 - 0.252 + 1.07] = -0.88 \quad , \quad (34)$$

where the terms inside the square brackets represent the relative size of the order p^0 , p^2 , and p^3 contributions, respectively. A similar pattern holds for the other octet axial vector matrix elements. Far from improving the convergence behavior of the octet-only chiral expansion of the axial currents, inclusion of recoil corrections makes it worse.

In order to explore further why the chiral corrections through $\mathcal{O}(p^2)$ are so significant, we collect the values of loop integral functions in Eq. (16) in Table IX. First, we note that the contribution of the vertex correction from Fig. 2a is suppressed due to its small coefficients β_{ij}^X , which are cubic functions of D, F . Although the coefficients $\lambda_{ij}\alpha_{ij}$ are also cubic in D, F , the coefficients of λ_{ij} are big as can clearly seen in Table I. Consequently, wavefunction renormalization has a significant impact. Moreover, the contributions from the self-energy, tadpole, and seagull diagrams all have the same sign as the the tree level axial couplings. These contributions add constructively. In addition, the coefficients of tadpole diagram and seagull diagrams are linear function of D, F , so they are enhanced relative to the other loops in this respect.

The relative size of the recoil corrections requires further explanation. To illustrate, consider the seagull contributions. Naïvely, the latter ought to be suppressed by roughly $m_k/m_N \sim 1/2$ relative to the $\mathcal{O}(p^2)$ loop effects. However, the presence of the $\arccos(-\mu/2)$ in these loops generates an additional numerical factor of π for these contributions at leading order in the $1/m_N$ expansion. It is both the large size of the kaon mass and this numerical

factor which are responsible for the large size of the $\mathcal{O}(p^3)$ effects. Moreover, such numerical enhancement factors appear at higher orders as well. For example, the $\mathcal{O}(p^5)$ contributions generated by wavefunction renormalization are also proportional to $\arccos(-\mu/2)$. Thus, we would expect the pattern shown in Eq. (33) to persist to higher orders^{#1}.

Finally, we illustrate the practical consequences of axial vector non-convergence by considering the strange quark contribution to the nucleon's spin, Δs . As shown in Ref. [11], one may express Δs in terms of the polarized structure function integrals

$$\Gamma_{p,n} = \int_0^1 dx g_1^{p,n}(x) \quad (35)$$

as

$$\Delta s = \frac{3}{2}[\Gamma_p + \Gamma_n] - \frac{5\sqrt{3}}{6}g_8^A \quad (36)$$

where g_8^A is the axial vector coupling associated with the matrix element $\langle p|J_\mu^8|p\rangle$. The combinations of LEC's required for this matrix element are

$$\alpha_{pp}^8 = \frac{1}{2\sqrt{3}}(3F - D) \quad (37)$$

$$\beta_{pp}^{8,K} = \frac{1}{\sqrt{3}}\left(\frac{2}{3}D^3 - 2D^2F\right) \quad (38)$$

$$\beta_{pp}^{8,\eta} = \frac{1}{24\sqrt{3}}(3F - D)^3 \quad (39)$$

$$\bar{\alpha}_{pp}^8 = \frac{1}{\sqrt{3}}\left(\frac{1}{2}d_2 - 2d_4\right) \quad (40)$$

$$\gamma_{pp}^{8,K} = -\frac{3}{2}, \quad \gamma_{pp}^{8,\eta} = 0 \quad (41)$$

$$\theta_{pp}^{8,K,\eta} = -4\gamma_{pp}^{8,K,\eta} \quad (42)$$

Using our results in scheme C, we obtain

$$g_8^A = 0.11[1 + 0.55 + 1] \quad (43)$$

where the terms in the brackets correspond to the order p^0 , p^2 , and p^3 contributions. Using this result and the world average data for the $\Gamma_{p,n}$, we obtain

^{#1}The problem of the large kaon mass in SU(3) CPT with baryons has also been addressed in Ref. [10].

$$\Delta s = 0.14 - 0.16[1 + 0.55 + 1] \tag{44}$$

where we have omitted the experimental error bars in the first term taken from the polarized deep inelastic scattering (DIS) data. The second term represents the contribution from g_8^A , broken by successive orders as above. We do not quote a total for Δs given that the $\mathcal{O}(p^3)$ contribution from g_8^A is as large as both the $\mathcal{O}(p^0)$ term as well as the first term on the RHS of Eq. (44). Given the poor convergence behavior of the expansion of g_8^A , extraction of Δs from polarized DIS data is problematic. In contrast, extractions of Δs from semi-inclusive measurements performed by the Hermes collaboration or elastic neutrino-nucleon scattering are not plagued by large SU(3)-breaking uncertainties. Whether inclusion of decuplet intermediate states reduces these SU(3)-breaking uncertainties requires further study.

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Figure Captions

FIG 1. Feynman diagrams for the wave function renormalization. The dashed and solid line denotes the pseudoscalar meson and baryon respectively.

FIG 2. The loop diagrams for the chiral corrections to the axial charge. The filled circle is the insertion of the axial current in Eq. (16).

TABLES

	kaon loop	η loop	π loop
λ_{pn}	$\frac{10}{3}D^2 - 4DF + 6F^2$	$\frac{1}{3}D^2 - 2DF + 3F^2$	$3(D + F)^2$
$\lambda_{\Lambda\Sigma^-}$	$\frac{8}{3}D^2 + 8F^2$	$\frac{4}{3}D^2$	$\frac{8}{3}D^2 + 4F^2$
$\lambda_{\Xi^0\Xi^-}$	$\frac{10}{3}D^2 + 4DF + 6F^2$	$\frac{1}{3}D^2 + 2DF + 3F^2$	$3(D - F)^2$
$\lambda_{p\Lambda}$	$\frac{7}{3}D^2 - 2DF + 9F^2$	$\frac{5}{6}D^2 - DF + \frac{3}{2}F^2$	$\frac{7}{2}D^2 + 3DF + \frac{3}{2}F^2$
$\lambda_{\Lambda\Sigma^-}$	$\frac{7}{3}D^2 + 2DF + 9F^2$	$\frac{5}{6}D^2 + DF + \frac{3}{2}F^2$	$\frac{7}{2}D^2 - 3DF + \frac{3}{2}F^2$
$\lambda_{n\Sigma^-}$	$\frac{11}{3}D^2 - 2DF + 5F^2$	$\frac{5}{6}D^2 - DF + \frac{3}{2}F^2$	$\frac{13}{6}D^2 + 3DF + \frac{11}{2}F^2$
$\lambda_{\Sigma^0\Xi^-}$	$\frac{11}{3}D^2 + 2DF + 5F^2$	$\frac{5}{6}D^2 + DF + \frac{3}{2}F^2$	$\frac{13}{6}D^2 - 3DF + \frac{11}{2}F^2$
λ_{pp}	$\frac{10}{3}D^2 - 4DF + 6F^2$	$\frac{1}{3}D^2 - 2DF + 3F^2$	$3(D + F)^2$
$\lambda_{\Lambda\Lambda}$	$\frac{4}{3}D^2 + 12F^2$	$\frac{4}{3}D^2$	$4D^2$
$\lambda_{\Sigma\Sigma}$	$4D^2 + 4F^2$	$\frac{4}{3}D^2$	$\frac{4}{3}D^2 + 8F^2$
$\lambda_{\Xi\Xi}$	$\frac{10}{3}D^2 + 4DF + 6F^2$	$\frac{1}{3}D^2 + 2DF + 3F^2$	$3(D - F)^2$

TABLE I. The coefficients λ_{ij}^X for the wave function renormalization.

	kaon loop	η loop	π loop
β_{pn}	$\frac{-D^3 + D^2F - 3DF^2 + 3F^3}{3}$	$\frac{D^3 - 5D^2F + 3DF^2 + 9F^3}{12}$	$\frac{1}{4}(D + F)^3$
$\beta_{\Lambda\Sigma^-}$	$-\frac{1}{\sqrt{6}}D^3 + \frac{1}{\sqrt{6}}DF^2$	$-\frac{\sqrt{6}}{9}D^3$	$\frac{2}{\sqrt{6}}D(\frac{D^2}{3} - 2F^2)$
$\beta_{\Xi^0\Xi^-}$	$\frac{-D^3 + D^2F + 3DF^2 + 3F^3}{3}$	$\frac{D^3 + 5D^2F + 3DF^2 - 9F^3}{12}$	$\frac{1}{4}(D - F)^3$
$\beta_{p\Lambda}$	$\frac{-5D^3 + 15D^2F + 9DF^2 - 27F^3}{6\sqrt{6}}$	$\frac{1}{\sqrt{6}}[-\frac{1}{6}D^3 + \frac{3}{2}DF^2]$	$\frac{\sqrt{6}}{4}D(D^2 - F^2)$
$\beta_{\Lambda\Sigma^-}$	$\frac{-5D^3 - 15D^2F + 9DF^2 + 27F^3}{6\sqrt{6}}$	$\frac{1}{\sqrt{6}}[-\frac{1}{6}D^3 + \frac{3}{2}DF^2]$	$\frac{\sqrt{6}}{4}D(D^2 - F^2)$
$\beta_{n\Sigma^-}$	$-\frac{D^3 + D^2F + 3DF^2 + 3F^3}{6}$	$-\frac{1}{6}D^3 + \frac{2}{3}D^2F - \frac{1}{2}DF^2$	$-\frac{D^3 - 2D^2F + 3DF^2 + 6F^3}{6}$
$\beta_{\Sigma^0\Xi^-}$	$\frac{-D^3 + D^2F - 3DF^2 + 3F^3}{6\sqrt{2}}$	$-\frac{D^3 + 4D^2F + 3DF^2}{6\sqrt{2}}$	$-\frac{D^3 + 2D^2F + 3DF^2 - 6F^3}{6\sqrt{2}}$

TABLE II. The coefficients β_{ij}^X for the vertex correction of Fig. 2a.

	kaon loop	η loop	π loop
γ_{pn}	$-\frac{1}{2}$	0	-1
$\gamma_{\Lambda\Sigma^-}$	$-\frac{1}{2}$	0	-1
$\gamma_{\Xi^0\Xi^-}$	$-\frac{1}{2}$	0	-1
$\gamma_{p\Lambda}$	$-\frac{3}{4}$	$-\frac{3}{8}$	$-\frac{3}{8}$
$\gamma_{\Lambda\Xi^-}$	$-\frac{3}{4}$	$-\frac{3}{8}$	$-\frac{3}{8}$
$\gamma_{n\Sigma^-}$	$-\frac{3}{4}$	$-\frac{3}{8}$	$-\frac{3}{8}$
$\gamma_{\Sigma^0\Xi^-}$	$-\frac{3}{4}$	$-\frac{3}{8}$	$-\frac{3}{8}$

TABLE III. The coefficients γ_{ij}^X for the tadpole diagram Fig. 2b.

	Experimental data	tree level fit
g_{pn}^A	(1.2573 ± 0.0028)	1.253
$g_{\Lambda\Sigma^-}^A$	$\sqrt{\frac{2}{3}}(0.742 \pm 0.018)$	0.64
$g_{\Xi^0\Xi^-}^A{}^\dagger$	—	0.31
$g_{p\Lambda}^A$	$-\sqrt{\frac{3}{2}}(0.718 \pm 0.015)$	-0.90
$g_{\Lambda\Xi^-}^A$	$\sqrt{\frac{3}{2}}(0.25 \pm 0.05)$	0.26
$g_{n\Sigma^-}^A$	(0.340 ± 0.017)	0.31
$g_{\Sigma^0\Xi^-}^A$	$\frac{1}{\sqrt{2}}(1.278 \pm 0.158)$	0.89
D	—	0.78
F	—	0.47
F/D	—	0.60
χ^2	—	0.1

TABLE IV. Experimental data and $\mathcal{O}(p^0)$ fits for the axial charge from hyperon semileptonix decays. The value in the bracket is the experimental value of g_1/f_1 . The channel with † is the prediction.

	Data	Tree level fit	One loop $\mathcal{O}(p^2)$ Fit B	One loop $\mathcal{O}(p^3)$ Fit B
g_{pn}^A	(1.2573 ± 0.0028)	1.253	1.245	1.25
$g_{\Lambda\Sigma^-}^A$	$\sqrt{\frac{2}{3}}(0.742 \pm 0.018)$	0.64	0.60	0.62
$g_{\Xi^0\Xi^-}^A{}^\dagger$	—	0.31	0.126	0.29
$g_{p\Lambda}^A$	$-\sqrt{\frac{3}{2}}(0.718 \pm 0.015)$	-0.90	-0.90	-0.89
$g_{\Lambda\Xi^-}^A$	$\sqrt{\frac{3}{2}}(0.25 \pm 0.05)$	0.26	0.31	0.30
$g_{n\Sigma^-}^A$	(0.340 ± 0.017)	0.31	0.35	0.34
$g_{\Sigma^0\Xi^-}^A$	$\frac{1}{\sqrt{2}}(1.278 \pm 0.158)$	0.89	0.905	0.90
D	—	0.78	0.55	0.41
F	—	0.47	0.41	0.26
F/D	—	0.60	0.75	0.63
χ^2	—	0.1	0.01	0.022
d_1	—	—	-1.08	-2.75
d_2	—	—	0.505	0.88
d_3	—	—	-0.574	-0.65
d_4	—	—	0.82	-0.11

TABLE V. Our fit with Scheme B up to $\mathcal{O}(p^2)$, $\mathcal{O}(p^3)$. The channel with † is the prediction.

	Full fit results	Tree level only	$\mathcal{O}(p^2)$ only	$\mathcal{O}(p^3)$ only
g_{pn}^A	1.25	0.671	0.245	0.334
$g_{\Lambda\Sigma^-}^A$	0.62	0.33	0.079	0.211
$g_{\Xi^0\Xi^-}^A{}^\dagger$	0.29	0.143	-0.031	0.178
$g_{p\Lambda}^A$	-0.89	-0.489	0.123	-0.524
$g_{\Lambda\Xi^-}^A$	0.30	0.157	-0.063	0.206
$g_{n\Sigma^-}^A$	0.34	0.143	0.053	0.144
$g_{\Sigma^0\Xi^-}^A$	0.90	0.474	-0.158	0.584

TABLE VI. The separation of our full up to $\mathcal{O}(p^3)$ fit results with Scheme B into tree level, pure $\mathcal{O}(p^2)$, and $\mathcal{O}(p^3)$ pieces for the sake of the discussion of convergence of the chiral expansion.

	Data	Tree level fit	One loop $\mathcal{O}(p^2)$ Fit C	One loop $\mathcal{O}(p^3)$ Fit C
g_{pn}^A	(1.2573 ± 0.0028)	1.253	1.26	1.26
$g_{\Lambda\Sigma^-}^A$	$\sqrt{\frac{2}{3}}(0.742 \pm 0.018)$	0.64	0.64	0.61
$g_{\Xi^0\Xi^-}^A$ [†]	—	0.31	0.24	0.22
$g_{p\Lambda}^A$	$-\sqrt{\frac{3}{2}}(0.718 \pm 0.015)$	-0.90	-0.85	-0.88
$g_{\Lambda\Xi^-}^A$	$\sqrt{\frac{3}{2}}(0.25 \pm 0.05)$	0.26	0.34	0.30
$g_{n\Sigma^-}^A$	(0.340 ± 0.017)	0.31	0.35	0.34
$g_{\Sigma^0\Xi^-}^A$	$\frac{1}{\sqrt{2}}(1.278 \pm 0.158)$	0.89	0.88	0.90
D	—	0.78	0.513	0.39
F	—	0.47	0.370	0.26
F/D	—	0.60	0.72	0.67
χ^2	—	0.1	0.004	0.013
d_1	—	—	-1.17	-2.73
d_2	—	—	0.60	0.82
d_3	—	—	-0.62	-0.54
d_4	—	—	0.84	0.094

TABLE VII. Our fit with Scheme C up to $\mathcal{O}(p^2)$, $\mathcal{O}(p^3)$. The channel with [†] is the prediction.

	Full fit results	Tree level only	$\mathcal{O}(p^2)$ only	$\mathcal{O}(p^3)$ only
g_{pn}^A	1.26	0.658	0.276	0.326
$g_{\Lambda\Sigma^-}^A$	0.61	0.318	0.095	0.197
$g_{\Xi^0\Xi^-}^A$ [†]	0.22	0.12	-0.042	0.142
$g_{p\Lambda}^A$	-0.88	-0.488	0.136	-0.528
$g_{\Lambda\Xi^-}^A$	0.30	0.17	-0.089	0.219
$g_{n\Sigma^-}^A$	0.34	0.12	0.101	0.119
$g_{\Sigma^0\Xi^-}^A$	0.90	0.465	-0.135	0.57

TABLE VIII. The separation of our full up to $\mathcal{O}(p^3)$ fit results with Scheme C into tree level, pure $\mathcal{O}(p^2)$, and $\mathcal{O}(p^3)$ pieces.

	kaon loop	η loop
I_a	0.21	0.27
I_b	-0.23	-0.237
I_c	0.167	0.230
I_d	0.34	0.424
J_A	0.533	0.504
Δ	-0.23	-0.237
$I(m_N^2)$	-1.37	-1.53

TABLE IX. The values of loop integral functions in Eq. (16).



